09/04/13

Unit - 5

Slope Stability

Introduction:

↳ Earth embankement are commonly required for railway, earth dam and river draining works.

↳ The stability of the embankement or slope on they are commonly called, should be very throughly analysed since their failure may lead loss of human life as well as economic loss.

↳ The failure of slopes takes places mainly due to

2m

1. The absence of gravitational force
2. Seepage force with in the soil
3. Excavation process
4. Gradual disintegration of the structure of the soil.

Slopes may be of two types

1. Infinite Slope
2. Finite Slope

Infinite Slope:

If a slope represents the boundary surface of a semi-infinite soil mass and the soil properties for all identical depth below the surface are constant is called as infinite slope and it is also known as unlimited

Finite Slope :

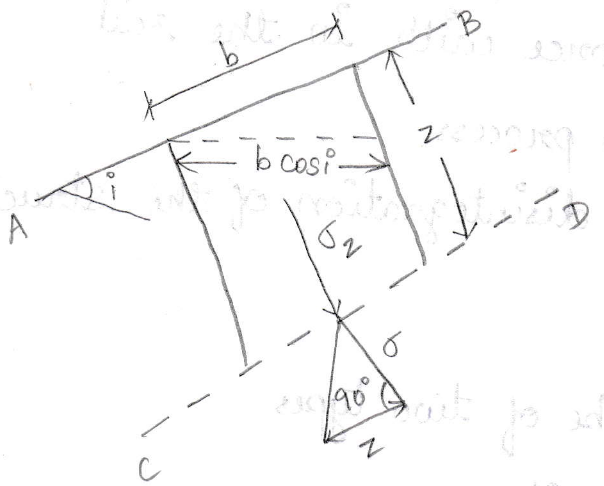
If the slope is limited extent it is called finite slope.

eg : Inclined faces of earthdam
Embankment and cuts etc..

Analysis of stability slope consist of two parts :

1. The determination of the shearing stress
2. The determination of shearing stress along this surface.

Stability Analysis of infinite slope :



↳ Figure shows an infinite slope AB inclined at angle i to the horizontal.

↳ Let CD represents the failure plane at depth 'z' below the surface.

↳ Consider a prism of soil of inclined length 'b' along the slope, at the any depth 'z' upto the critical surface.

↳ Horizontal length of prism = $b \cos i$

Volume of prism = $z\,b \cos i$

Weight of prism (W) = $\gamma . z\, b \cos i$

[On Surface = γ_{dry}

Middle = $\gamma_{saturation}$

Water table = $\gamma_{submerge}$]

Vertical stress (σ_z) on the surface CD is given by,

$$\sigma_z = \frac{W}{b}$$

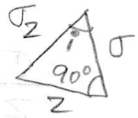
$$= \frac{\gamma z b \cos i}{b}$$

$$\therefore \sigma_z = \gamma z \cos i$$

↳ If σ and τ are stress components normal and tangential to the surface CD, we have

$$\tau = \sigma_z \sin i \qquad \sin i = \frac{\tau}{\sigma_z}$$

$$\sigma = \sigma_z \cos i \qquad \cos i = \frac{\sigma}{\sigma_z}$$



$$\sigma = \gamma z \cos i . \cos i = \gamma z \cos^2 i \longrightarrow (1)$$

$$\tau = \gamma z \cos i \sin i \longrightarrow (2)$$

↳ Factor of safety of the slope against sliding due to shear is given by

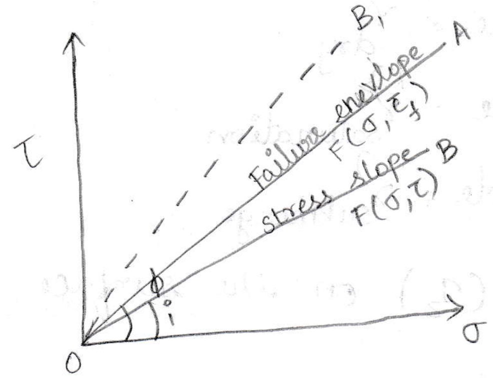
$$\boxed{F = \frac{\tau_f}{\tau} = \frac{\text{Shear Strength}}{\text{Shear Stress}}}$$

Consider the two cases of soil

i) Cohesion less soil

ii) Cohesive soil

CASE : 1 Cohesion less soil $(c=0)$



In figure OA is the failure envelope for cohesionless soil.

By definition,

$$\boxed{\tau_f = \sigma \tan\phi}$$

Given slope i, σ and τ vary with z

$$\frac{\sigma}{\tau} = \frac{\cos i}{\sin i} = \cot i = \text{constant}$$

Line OB drawn at inclination i, with σ-axis

$$\sigma = \tau \cot i$$

$$\tau = \sigma \tan i$$

$$F = \frac{\tau_f}{\tau}$$

$$= \frac{\sigma \tan\phi}{\sigma \tan i}$$

$$\therefore F = \frac{\tan\phi}{\tan i}$$

$$\boxed{F = \frac{\tau_f}{\tau} = \frac{\tan\phi}{\tan i}}$$

Submerged soil:

If the slope is submerged γ replaced by γ'.

From equation ① and ②,

$$\sigma = \gamma' z \cos^2 i$$

$$\tau = \gamma' z \cos i \sin i$$

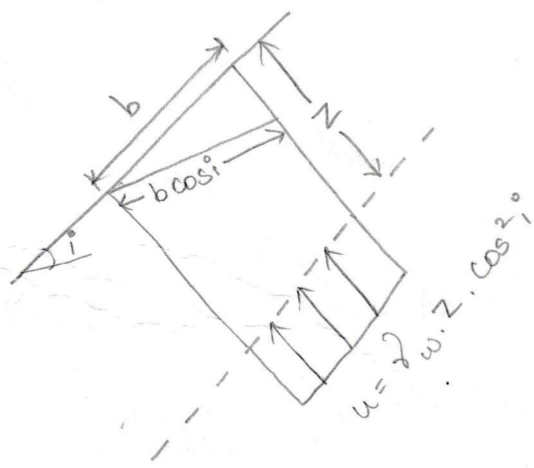
$$F = \frac{\tau_f}{\tau}$$

$$= \frac{\sigma \tan\phi}{\gamma' z \cos i \sin i}$$

$$= \frac{\cancel{\gamma'} \cancel{z} \cos^{\cancel{2}} i \tan\phi}{\cancel{\gamma'} \cancel{z} \cancel{\cos i} \sin i}$$

$$\therefore \boxed{F = \frac{\tan\phi}{\tan i}}$$

Seepage along the slope: (water table at the surface)



From equation ① and ②,

$$\sigma = \gamma_{sat} z \cos^2 i$$

$$\tau = \gamma_{sat} z \cos i \cdot \sin i$$

In addition, upward 'u' due to seeping water

$$u = \gamma_w \cdot z \cos^2 i$$

Hence, Effective stress $(\sigma') = \sigma - u$

$$\sigma' = \gamma_{sat} z \cos^2 i - \gamma_w z \cos^2 i$$

$$= z\cos^2 i(\gamma_{sat} - \gamma_w)$$

$$\therefore \sigma' = \gamma' z \cos^2 i$$

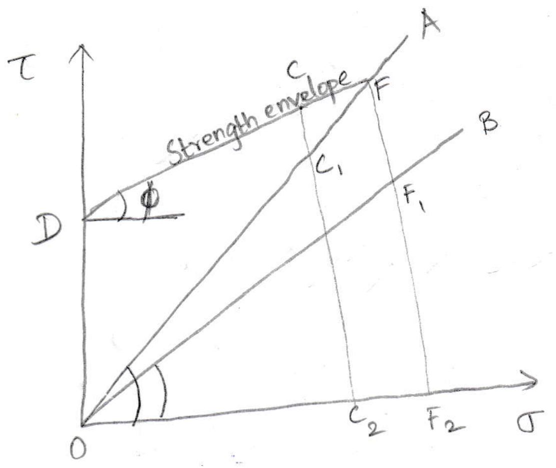
$$F = \frac{\tau_f}{\tau}$$

$$= \frac{\sigma' \tan\phi}{\tau}$$

$$= \frac{\gamma' \cancel{z} \cos^{\cancel{2}} i \tan\phi}{\gamma_{sat} \cancel{z} \cancel{\cos i} \sin i}$$

$$\therefore \boxed{F = \frac{\gamma' \tan\phi}{\gamma_{sat} \tan i}}$$

CASE : 2 Cohesive soil $(\phi = 0)$



From the figure,

DA = strength envelope

By definition,

$$\tau_f = c + \sigma \tan\phi$$

$i \leq \phi$ = slope stable

$i > \phi$ = the slope can be stable upto a limited depth is known as critical depth (H_c). ($F = 1$)

From equation ① and ②,

$$\sigma = \gamma z \cos^2 i$$

$$\tau = \gamma z \cos i \sin i$$

$$F = \frac{\tau_f}{\tau}$$

$$= \frac{c + \sigma \tan\phi}{\tau}$$

$$= \frac{c + \gamma z \cos^2 i \tan\phi}{\gamma z \cos i \sin i}$$

$$\therefore \boxed{F = \frac{c}{\gamma z \cos i \sin i} + \frac{\tan\phi}{\tan i}}$$

For the critical depth $z = H_c$

i.e., $F = 1$

WKT,

$$F = \frac{\tau_f}{\tau}$$

$$\tau = \tau_f$$

$$\gamma H_c \cos i \sin i = c + \gamma H_c \cos^2 i \tan\phi$$

$$c = \gamma H_c \cos i \sin i - \gamma H_c \cos^2 i \tan\phi$$

$$c = \gamma H_c (\cos i \sin i - \cos^2 i \tan\phi)$$

$$\gamma H_c = \frac{c}{\cos i \sin i - \cos^2 i \tan\phi}$$

$$= \frac{c}{\cos^2 i \left[\frac{\cos i \sin i}{\cos^2 i} - \tan\phi\right]}$$

$$\gamma H_c = \frac{c}{(\tan i - \tan\phi)\cos^2 i}$$

$$(\tan i - \tan\phi)\cos^2 i = \frac{c}{\gamma H_c} \rightarrow (a)$$

$$\boxed{S_n = \frac{c}{\gamma H_c}}$$

where, S_n = stability number

Let, F_c is FOS with respect to cohesion

c_m is mobilised cohesion at depth 'H'

$$\boxed{c_m = \frac{c}{F_c}}$$

From stability number,

$$S_n = \frac{c}{\gamma H_c}$$

$$= \frac{c_m}{\gamma H}$$

$$S_n = \frac{c}{F_c \cdot \gamma H}$$

$$S_n = (\tan i - \tan\phi)\cos^2 i = \frac{c}{F_c \cdot \gamma H} \rightarrow (b)$$

Solving (a) and (b) equation,

$$\boxed{F_c = \frac{H_c}{H}}$$

where, F_c is FOS w.r to cohesion and height

10/04/13 Submerged slope :

$$F = \frac{\tau_f}{\tau} = \frac{c + \sigma \tan\phi}{\tau}$$

WKT,
$$\sigma = \gamma' z \cos^2 i$$
$$\tau = \gamma' z \cos i \sin i$$

$$F = \frac{c + \gamma' z \cos^2 i \tan\phi}{\gamma' z \cos i \sin i}$$

From equation ⓐ,

$$H_c = \frac{c}{\gamma'} \cdot \frac{1}{(\tan i - \tan\phi)\cos^2 i}$$

$$H_c = \frac{c}{\gamma'} \cdot \frac{\sec^2 i}{\tan i - \tan\phi}$$

Seepage along the slope:

$$F = \frac{c + \gamma' z \cos^2 i \tan\phi}{\gamma_{sat} z \cos i \sin i}$$

$$H_c = \frac{c}{(\gamma_{sat} \tan i - \gamma' \tan\phi)\cos^2 i}$$

Problem:

1. A long natural slope of cohession less soil is inclined at 12° to the horizontal, take $\phi = 30°$. Determine the FOS of the slope. If the slope is completely submerged then what will be change in the FOS.

Solution:

For dry soil, $i = 12^\circ$

$\phi = 30^\circ$

$$F = \frac{\tau_f}{\tau} = \frac{\tan\phi}{\tan i}$$

$$= \frac{\tan 30^\circ}{\tan 12^\circ}$$

$\therefore F = 2.72$

Effect of submerge,

$$F = \frac{\tau_f}{\tau} = \frac{\tan\phi}{\tan i}$$

$$= \frac{\tan 30^\circ}{\tan 12^\circ}$$

(FOS Same)

$\therefore F = 2.72$

2. A long natural slope of sandy soil is inclined at 10° to the horizontal. The water take is at the surface and the seepage is parallel to the slope. If the saturated unit weight of soil is $19.5\ kN/m^3$, Determine the FOS of slope. Take $\phi = 25^\circ$.

Given:

$i = 10^\circ$

$\phi = 25^\circ$

$\gamma_{sat} = 19.5\ kN/m^3$

Solution:

$$F = \frac{\gamma' \tan\phi}{\gamma_{sat} \tan i}$$

$$= \frac{(\gamma_{sat} - \gamma_w) \tan\phi}{\gamma_{sat} \tan i}$$

$$= \frac{(19.5 - 9.81) \tan 25^\circ}{19.5 \tan 10^\circ}$$

$\therefore F = 1.31$

3. A long natural slope in the $c-\phi$ soil is inclined at 12° to the horizontal. The water table is at the surface and seepage is parallel to the slope. If a plan slip has developed at a depth of 4m. Determine FOS of the slope.

Given:

$C = 8 \, kN/m^2$

$\phi = 22^\circ$

$\gamma_{sat} = 19 \, kN/m^3$

$i = 12^\circ$

$z = 4m$

Solution:

$$F = \frac{c + \gamma' z \cos^2 i \tan\phi}{\gamma_{sat} z \cos i \sin i}$$

$$= \frac{8 + (19 - 9.81) 4 \cos^2 12^\circ \tan 22^\circ}{19 \times 4 \cos 12^\circ \sin 12^\circ}$$

$\therefore F = 1.44$

Slope failure mechanism:

↳ Failure of finite slope occur along the surface which is a curve.

↳ Stability calculation, the curve representing the real surface of sliding is usually replaced by an arc of circle or logarithmic spiral.

Two basic type failure of finite slope

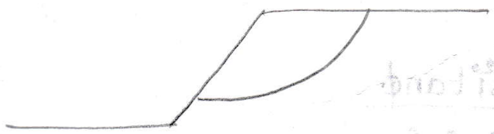
1. Slope failure
   - Toe failure
   - Face failure
2. Base failure

Slope failure:

If the failure occurs along the surface of sliding that intercept the slope at or above its toe, the slide is known as slope failure.

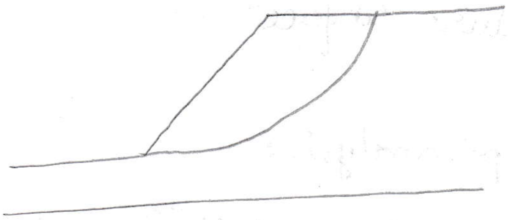
Face failure:

If the arc passes above the toe, this type of failure is called as face failure.



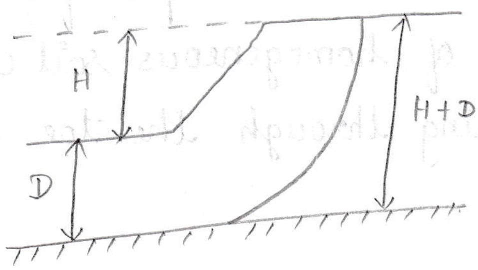
Toe failure :

If the arc passes through the toe, this type of failure is called as toe failure.



Base failure :

If the soil beneath the toe of the slope is weak the failure occurs along the surface that passes at some distance below the toes of the slope such a type of failure is called as base failure.



$$D_f = \frac{H+D}{H}$$

The ratio of the total depth (H+D) to depth (H) is called the depth factor (D_f).

For, Toe failure $D_f = 1$

Base failure $D_f > 1$

Type of slip surfaces or failure surface:

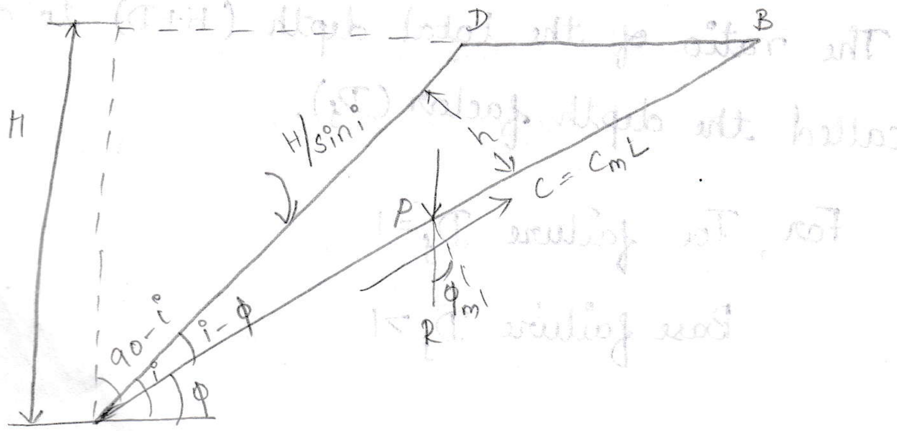
1. Planer failure surface
2. Circular failure surface
3. Non-circular failure surface

Method of finite slope analysis:

1. Culmann's method of planar failure surface.
2. The swedish circle method (slip circle method).
3. The friction circle method
4. Bishop's method.

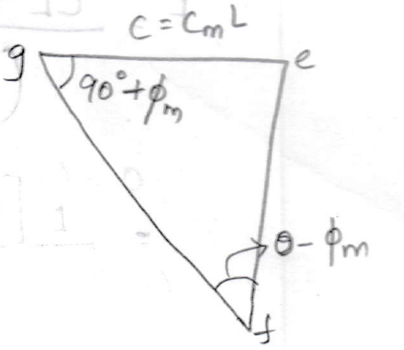
1. Culmann's Method:

Culmann's consider a simple failure mechanism of slope of homogeneous soil with plane failure surface passing through the toe of the slope.



culmann's slip plane

Let AB be the any probable slip plane. The soil wedge ApB is in equilibrium under the action of three forces.



Weight of soil wedge (W),

$$W = \frac{1}{2} AB.h.\gamma \quad (AB = L)$$

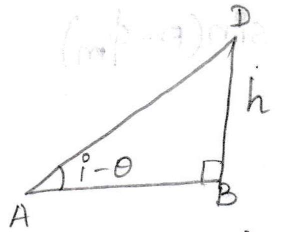
$$W = \frac{1}{2} Lh\gamma \longrightarrow ①$$

The cohessive force 'c' along the surface AB resisting motion = $C_m . L$

The reaction R, inclined at angle (ϕ_m) to the horizontal.

$$\frac{h}{\sin(i-\theta)} = \frac{H}{\sin i}$$

$$h = \frac{H \sin(i-\theta)}{\sin i}$$



$$\sin(i-\theta) = \frac{h}{AD}$$

$$AD = \frac{h}{\sin(i-\theta)}$$

Sub h value in ①,

$$W = \frac{1}{2} . L . \gamma . \frac{H \sin(i-\theta)}{\sin i} \longrightarrow ②$$

If c and ϕ are shear parameter,

Shear strength (τ_f) = $CL + W\cos\theta \tan\phi$
$(\sigma = W\cos\theta)$

The weight component parallel to the plane, causing sliding $\tau = W\sin\theta$

$$F = \frac{\tau_f}{\tau}$$

$$= \frac{CL + W\cos\theta\tan\phi}{W\sin\theta}$$

From equation ②,

$$F = \frac{CL + \left(\frac{1}{2} L\gamma \cdot \frac{H\sin(i-\theta)}{\sin i}\right)\cos\theta \tan\phi}{\left(\frac{1}{2} L\gamma \cdot \frac{H\sin(i-\theta)}{\sin i}\right)\sin\theta}$$

$$= \frac{L\left[C + \left(\frac{1}{2}\cdot\gamma \cdot \frac{H\sin(i-\theta)}{\sin i}\right)\cos\theta \tan\phi\right]}{\frac{1}{2}\cdot L\gamma \cdot \frac{H\sin(i-\theta)}{\sin i}\sin\theta}$$

$$F = \frac{C + \frac{1}{2}\cdot\gamma H \frac{\sin(i-\theta)}{\sin i}\cos\theta \tan\phi}{\frac{1}{2}\cdot\gamma H \cdot \frac{\sin(i-\theta)}{\sin i}\sin\theta} \longrightarrow ④$$

From force triangle,

$$\frac{C_m L}{\sin(\theta-\phi_m)} = \frac{W}{\sin(90^\circ+\phi_m)}$$

$$= \frac{W}{\cos\phi_m}$$

where, C_m = mobilised cohesion

ϕ_m = mobilised angle of internal friction

$$\frac{C_m L}{\sin(\theta-\phi_m)} = \frac{W}{\cos\phi_m} = \frac{\frac{1}{2} L\gamma H \sin(i-\theta)}{\sin i \cos\phi_m}$$

$$\frac{C_m}{\gamma H} = \frac{1}{2}\left[\frac{\sin(\theta-\phi_m)\sin(i-\theta)}{\sin i \cos\phi_m}\right]$$

$$\frac{C_m}{\gamma H} = \frac{1}{2}\operatorname{cosec} i \sec\phi_m \sin(\theta-\phi_m)\sin(i-\theta) \longrightarrow ⑤$$

$$\frac{C_m}{\gamma H} = S_n$$

Where, S_n = stability number

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For failure to occur, S_n has to be maximum,

when $\theta = \theta_c$, $\frac{d}{d\theta}(S_n) = 0$

$$\frac{d}{d\theta}(S_n) = \frac{d}{d\theta}\left[\sin(i-\theta)\sin(\theta-\phi_m)\right] = 0$$

$$\sin(i-\theta)\cos(\theta-\phi_m) - \sin(\theta-\phi_m)\cos(i-\theta) = 0$$

$$\sin(i-\theta)\cos(\theta-\phi_m) = \sin(\theta-\phi_m)\cos(i-\theta)$$

$$\frac{\sin(i-\theta)}{\cos(i-\theta)} = \frac{\sin(\theta-\phi_m)}{\cos(\theta-\phi_m)}$$

$$\tan(i-\theta) = \tan(\theta-\phi_m)$$

$$i-\theta = \theta-\phi_m$$

$$i+\phi_m = 2\theta$$

$$\theta = \frac{i+\phi_m}{2}$$

(or) $\qquad [\because \theta = \theta_c]$

$$\theta_c = \frac{i+\phi_m}{2}$$

θ_c is angle of inclination of critical slip plane.

Sub θ_c max in equ ⑤, we get

$$\left[\frac{C_m}{\gamma H}\right]_{max} = \frac{1}{2}\operatorname{cosec} i \sec\phi_m \sin\left(i - \frac{i+\phi_m}{2}\right)\sin\left(\frac{i+\phi_m}{2} - \phi_m\right)$$

$$= \frac{1}{2}\operatorname{cosec} i \sec\phi_m \sin\left(\frac{i+\phi_m}{2}\right)\sin\left(\frac{i-\phi_m}{2}\right)$$

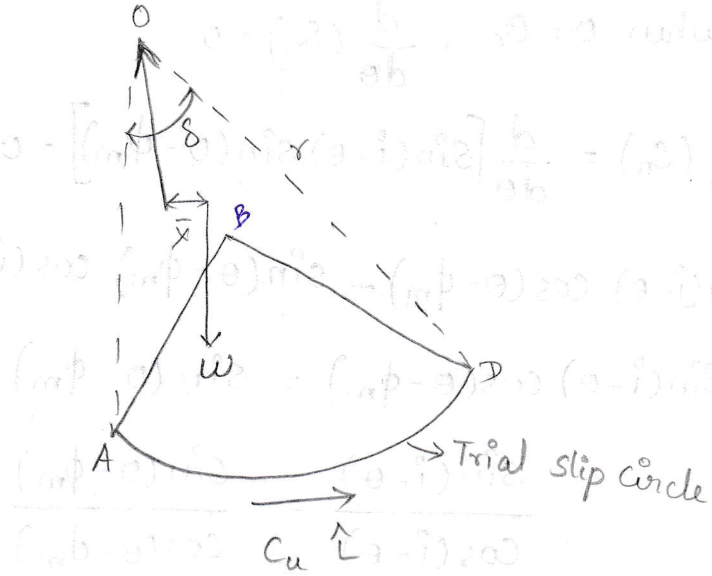
$$= \frac{1}{2}\operatorname{cosec} i \sec\phi_m \left[\frac{1-\cos(i-\phi_m)}{2}\right]$$

$$\frac{C_m}{\gamma H} = \frac{1-\cos(i-\phi_m)}{4\sin i\cos\phi_m}$$

$$H_{safe} = \frac{4C_m \sin i\cos\phi_m}{\gamma(1-\cos(i-\phi_m))}$$

8M

Swedish slip circle method or method of slices:



The method developed by swedish engineers, assume that the surface sliding is an arc of circle.

Case i): Analysis of purely cohessive soil ($\phi = 0$)

case ii): Analysis a soil cohessive both cohession and friction ($c - \phi$ analysis)

Let A, B be a slope, the stability of which is to be determine.

The method consisting assuming a number of trial slip circle and finding the FOS of each.

The circle corresponding to the minimum FOS is the ~~cir~~ Critical slip circle.

Let AD be a trial slip circle with 'r' as the radius and O as the center of rotation.

Let W be the weight of soil which ABDA acting true it centroid.

Driving moment $M_D = W.\bar{x}$

where, $\bar{x}$ is distance of line action 'w' from the Vertical line passing through the centre of rotation.

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The shear resistance develope along the slip surface will be $C_u \hat{L}$. It acts at a radial distance 'r' from the centre of rotation. Hence,

$$\text{Resisting moment } M_R = C_u \hat{L} r$$

where, C_u = Unit cohesion

$\hat{L}$ = Length of slip arc AD

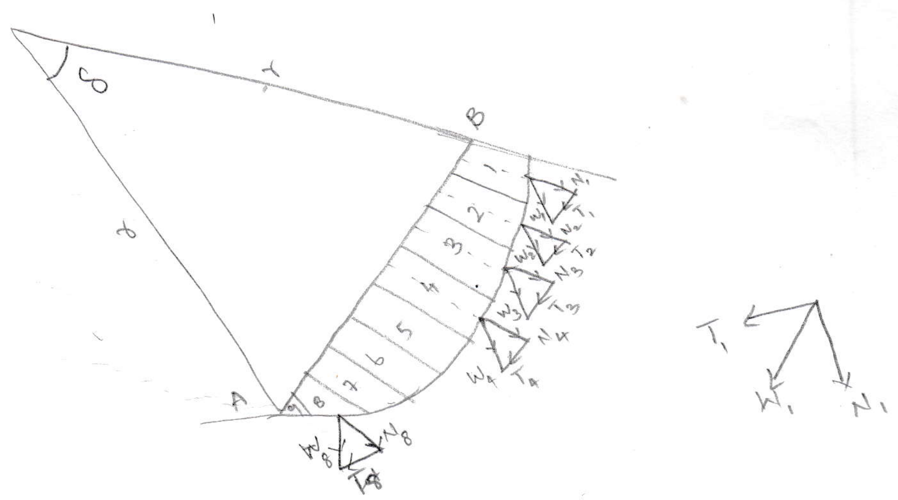
$$\hat{L} = \frac{2\pi r \delta}{360}$$

δ = Angle subtended by the arc AB at the center.

FOS against sliding is

$$F = \frac{M_R}{M_D} = \frac{C_u \hat{L} r}{W \bar{x}}$$

15/04/13 Case: ii C-ϕ Analysis



For the entire slip surface AB, we have

$$\text{Driving Moment } (M_D) = r \cdot \Sigma T$$

$$\text{Resisting Moment } (M_R) = r[C \, \Sigma \Delta L + \tan\phi \, \Sigma N]$$