M. Sowmiya.

UNIT - 4.

SHEAR STRENGTH :

The shear strength of a soil is the resistance to deformation by continuous, shear displacement of soil Particle or on Masses upon the action of a shear stress.

TWO FORMS :

Two forms of failure condition

i) shearing Properties.
ii) shearing resistance.
iii) shearing resistance of a soil.

component :

1) Structural resistance - Displacement of the soil. because of the interloking of the Particle.

2) Frictional resistance - Translocation between the individual soil Particle @ their contact Point.

3) Cohesion (or) adhesion - Between the surface of the soil Particle.

Small soil Particle Bonding.

theoretical consideration :

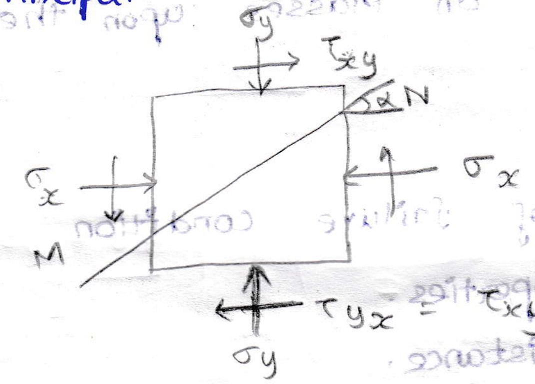
when soil mass is ~~known~~ loaded ~~small~~ more number of Planes Passes and stress component on each plane depend upon the direction of Plane.

Three typical Plane mutually orthogonal to each other :

i) stress is wholly normal and no ~~Know~~ a stress is Principal stress.

ii) normal stress acting on the Plane Principal stresses.

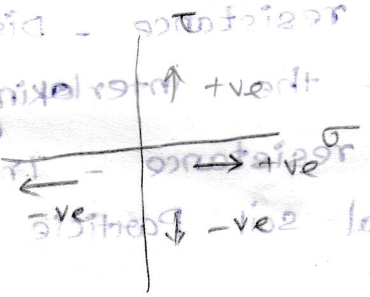
(iii) decreasing in Magnitude of the normal stress, this planes are called ~~neither~~ Major, intermediate and Minor Principal plane and corresponding normal stress on them are called Major Principal stress (σ_1), intermediate Principal stress (σ_2) and Minor principal stress (σ_3).

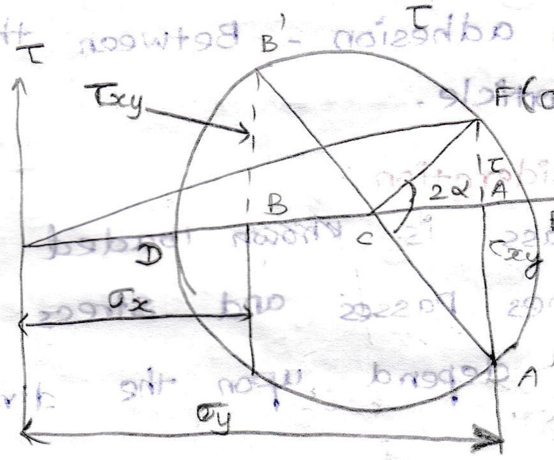


$\tau \rightarrow$ shear stress or tangential stress.

If stresses of soil sample.

Sign Conversion :





Mohr stress circle.

Expression for Normal Stress & shearing stress on any Plane MN @ any angle :-

$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \left[\frac{\sigma_y - \sigma_x}{2}\right]\cos 2\alpha + \tau_{xy}\sin 2\alpha \quad \rightarrow (1)$$

$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\alpha - \tau_{xy}\cos 2\alpha \quad \rightarrow (2)$$

where

σ_y = Normal stress on Plane y axis.

σ_x = Normal stress on Plane x axis

$\tau_{xy} = \tau_{yx}$ = Shear stress on two Planes.

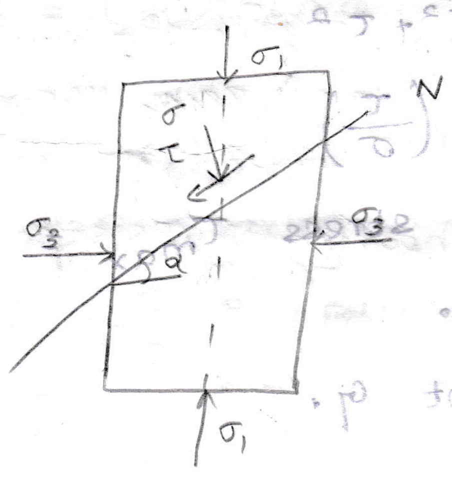
By adding (1) and (2).

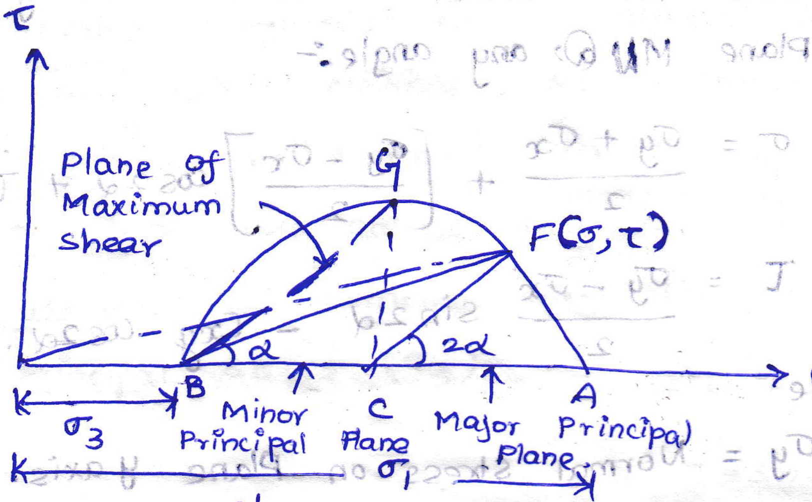
$$\left(\sigma - \frac{\sigma_y + \sigma_x}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2 \quad \rightarrow (3)$$

eq(3) is the equation of circle whose centre as a co-ordinate has

$$\left(\frac{\sigma_y + \sigma_x}{2}, 0\right)$$

Radius of $\sqrt{\left\{\frac{1}{2}(\sigma_y - \sigma_x)\right\}^2 + \tau_{xy}^2}$.



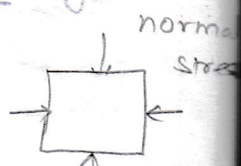


$$\sigma = \left(\frac{\sigma_y + \sigma_x}{2}\right) + \left(\frac{\sigma_y - \sigma_x}{2}\right)\cos 2\alpha + \tau_{xy} \sin 2\alpha$$

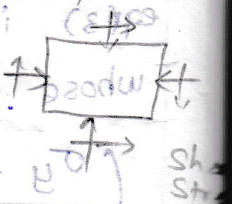
<u>x & y are inclined :-</u>

$$\sigma = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\alpha$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha$$



$$\tau = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha$$



<u>x & y are horizontal & vertical :-</u>

The Resultant stress @ any plane is

$\cancel{\sqrt{\sigma_1^2 + \sigma_3^2}}$. $\sqrt{\sigma^2 + \tau^2}$

$$\beta = \tan^{-1}\left(\frac{\tau}{\sigma}\right)$$

Maximum shear stress $\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2}$

when $\alpha = 45°$

τ_{max} @ Point G.

Mohr - coulomb failure theory :

1900

- Material fail essentially by shear ~~that~~ the critical shear stress causing failure depends upon the Properties of the Material as well as on the normal stress on the failure Plane.
- The ultimate strength of the Material is determine by stresses on the Potential failure Plane.
- When the Material is subjected to three dimensional Principal stress, the intermediate Principal stress does not have any influence on the strength of Material.

The theory can be expressed algebraically by the equation of

$$\tau_f = S = F(\sigma).$$

where $\tau_f = S$ = Shear stress on failure plane

$F(\sigma)$ = Function of normal stress.

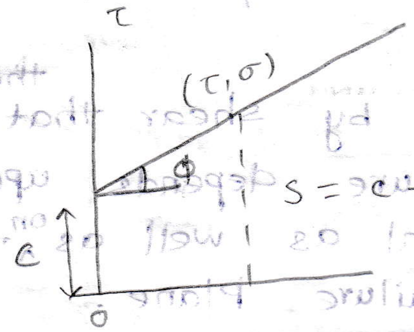
$$S = C + \sigma \tan \phi.$$

where

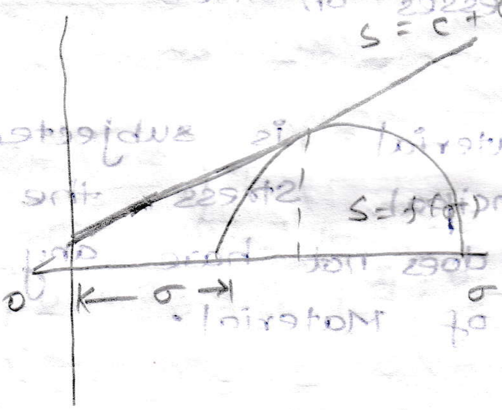
C = cohesive soil.

ϕ = shearing angle

$C - \phi$ = empirical constants.



coulumb's ~~constant~~ CONCEPT



MOHR CONCEPT

The normal and shear stress are plotted with the graph. we obtain a curve, ie, ~~shear~~ strength envelope.

The c and π are intercept on the shear axis and the slope of the straight line of equation $s = c + \sigma \tan\phi$.

This parameters are ~~easy~~ usefully termed as cohesion and angle of internal friction of shearing resistance.

8m

Effective Stress Principle :

$$\tau_f = c' + \sigma' \tan \phi'$$

$$\tau_f = c' + (\sigma - u) \tan \phi'.$$

where

c' = effective cohesion.

ϕ' = effective angle of shearing.

In terms total stress :

The eqn can be taken as,,

$$\tau_f = c_u + \sigma \tan \phi_u.$$

c_u = apparent cohesion.

ϕ_u = apparent angle of resistance.

σ' & τ' = included @ an angle α.

σ_1' & σ_3' = are effective Major & Minor principal stress :

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha.$$

$$\tau' = \left(\frac{\sigma_1 - \sigma_3}{2}\right) \sin 2\alpha.$$

Sub σ_1' in $\tau_f = c' + \sigma' \tan \phi'$

we get

$$\tau_f = c' + \tan \phi' \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha\right.$$

The Plane on which ~~radial~~ failure will take Place is one on which the different between the shear strength and shear stress is minimum?

$$(\tau_f - \tau)$$

$$(\tau_f - \tau) = c' + \frac{\sigma_1' + \sigma_3'}{2} \tan\phi' + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \tan\phi' - \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha.$$

Differentiating this with respect to α we get

$$\frac{d}{d\alpha}(\tau_f - \tau) = -(\sigma_1' - \sigma_3') \sin 2\alpha \tan\phi' - (\sigma_1' - \sigma_3') \cos 2\alpha$$

for a minimum

$$(\tau_f - \tau), \frac{d}{d\alpha}(\tau_f - \tau) = 0$$

This gives

$$\cos 2\alpha = - \sin 2\alpha \tan\phi'$$

(or).

$$\cot 2\alpha = - \tan\phi'$$

$$= \cot (90 + \phi)$$

$$\boxed{\alpha = \alpha_f = 45^\circ + \frac{\phi'}{2}}.$$

The above expression for the location of the failure plane can be directly derived from the Mohr circle

$$\boxed{2\alpha = 90^\circ + \phi'}$$

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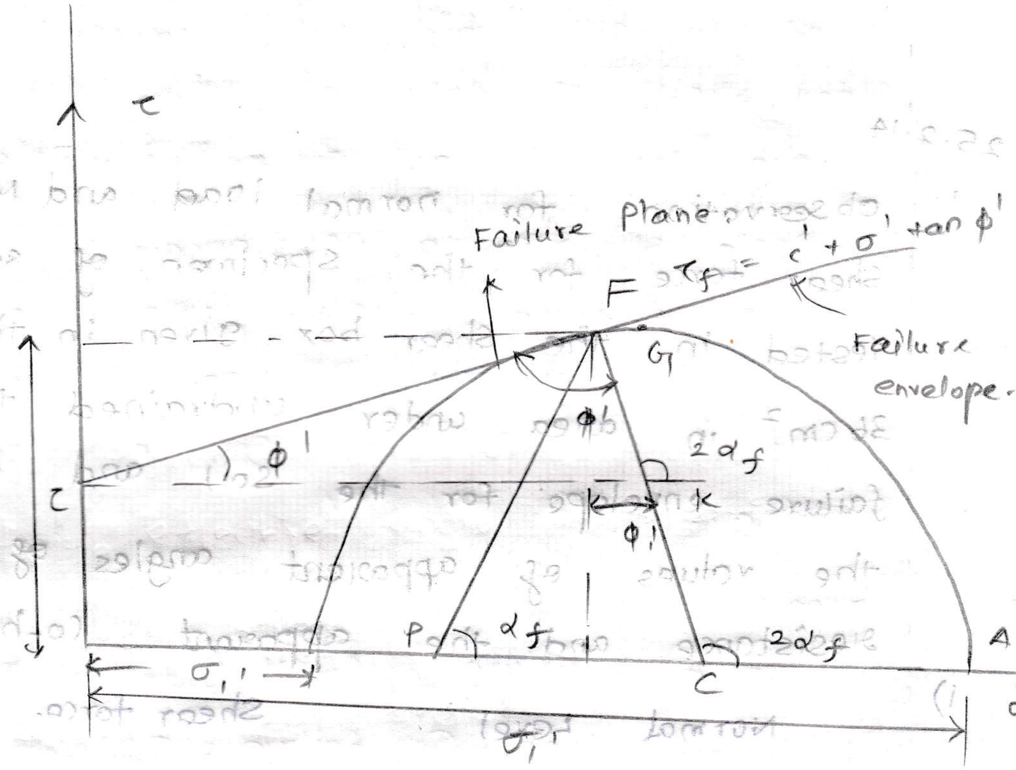


<u>Measurement of shear strain</u> :-

It can be determine by laboratory test

i) Direct shear stress.

ii) Triaxial shear stress.

iii) unconfined compression.

iv) ~~Tran~~ vane shear stress.

<u>Depending upon Drainage condition</u> :-

- undrained Test or Quick Test.
- consolidated undrained Test.
- Drained Test.

undrained Test :(coming out of water)

25.2.14

1. observation for normal load and Maximum shear force for the specimen of sandy clay tested in the shear box given in the table, 36 cm^2 in area under undrained test. Plot the failure envelope for the soil and determine the values of apparent angles of shearing resistance and the apparent co-hesion.

1)

Normal Level	Shear force.
100	110
200	152
300	193
400	235

To find :

Angle of shearing stress $\phi_u = ?$

cohesion $c_u = ?$

~~soln~~

2. The samples of compacted soil, clean dry sand that tested in a shear box 6cm × 6cm and the following results were obtained

~~6cm × 6cm~~

Normal load (N)	Densest Peak shear load	loosest Ultimate shear load
100	90	80
200	181	158
300	270	235
400	362	300

Maximum
sandy clay
the table,
test. ~~Plot~~ th
determine
of shearing
co-hesion.

cp.

$\phi_u = ?$

$c_u = ?$

dry
oil), clean ~~dyarn~~

hear box 6cm x

were obtained

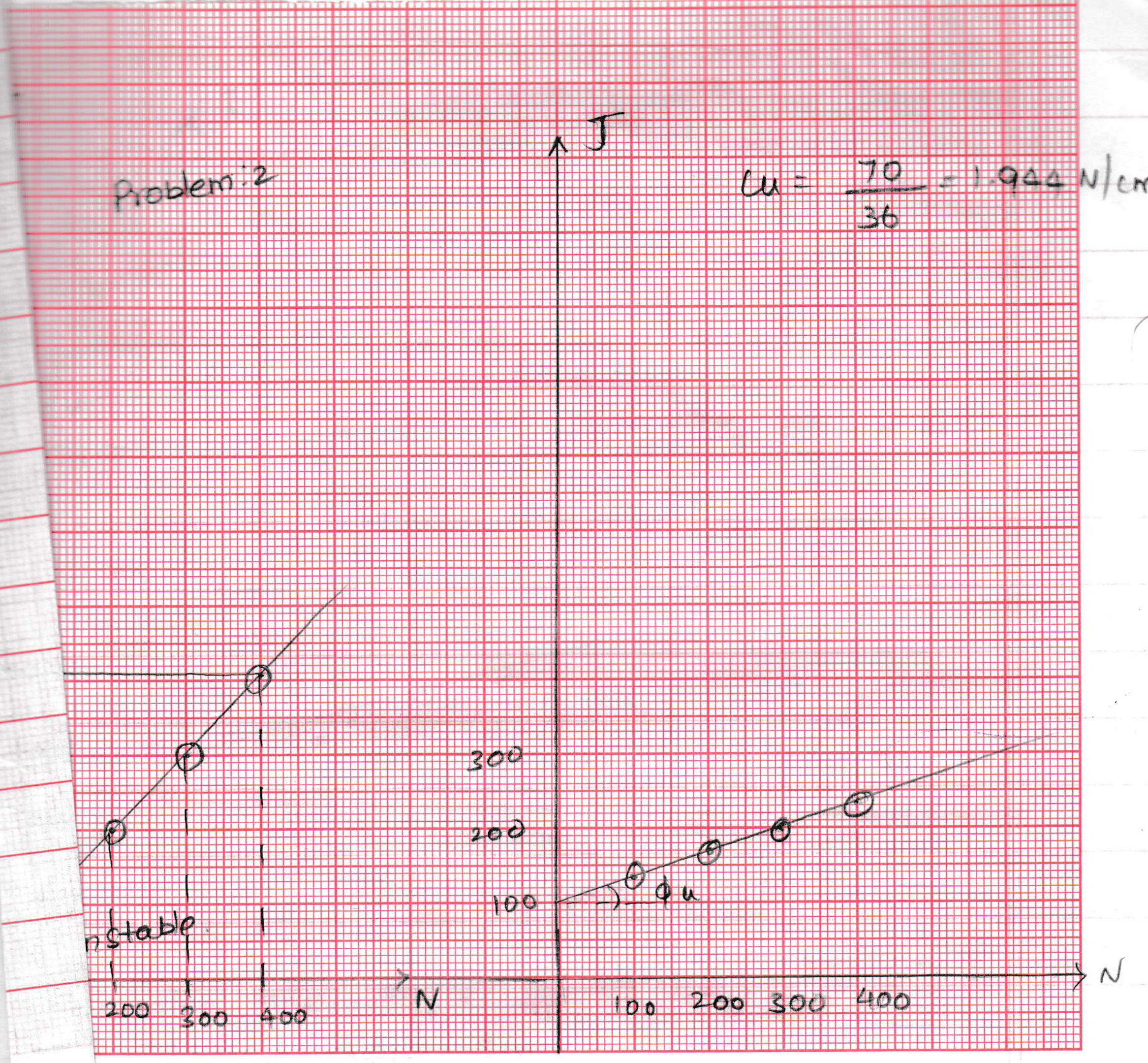
loosest
ultimate
shear load

80

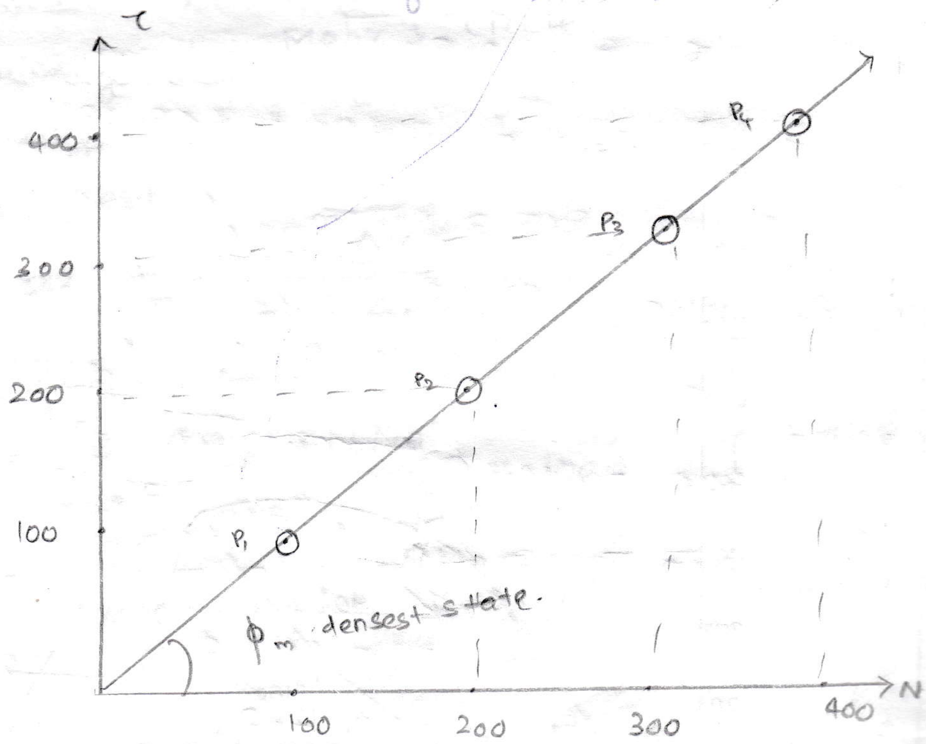
158

235

300



ii) loosest state.



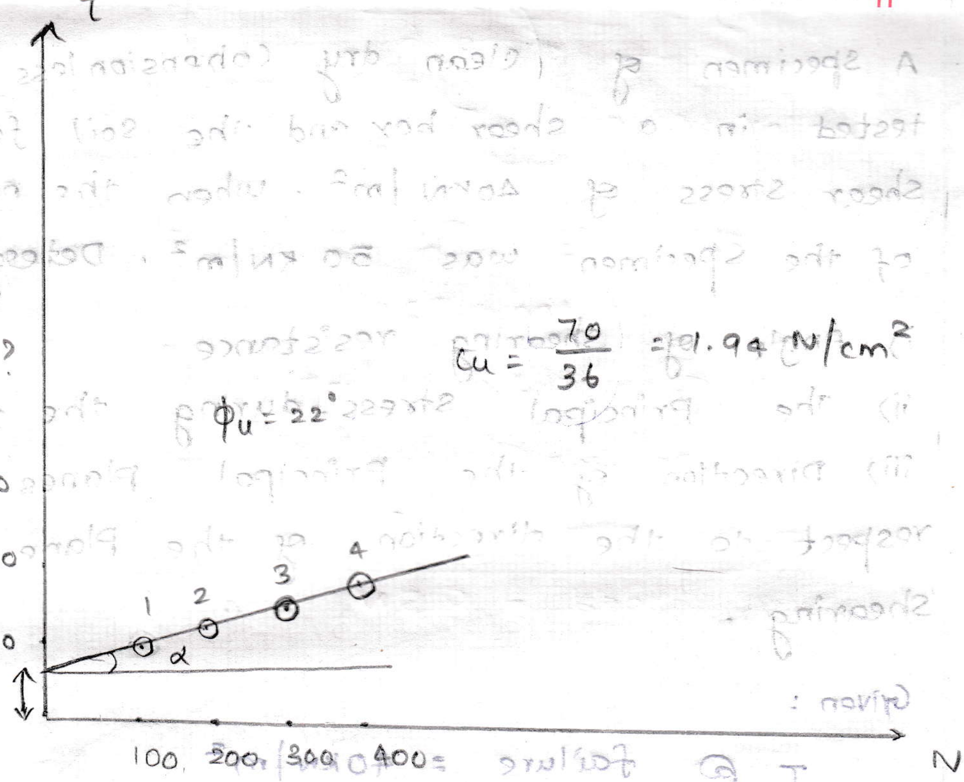
$\phi_u = ?$

$c_u = ?$

$c_u = \frac{70}{36} = 1.94 \ N/cm^2$

$\phi_u = 22^\circ$

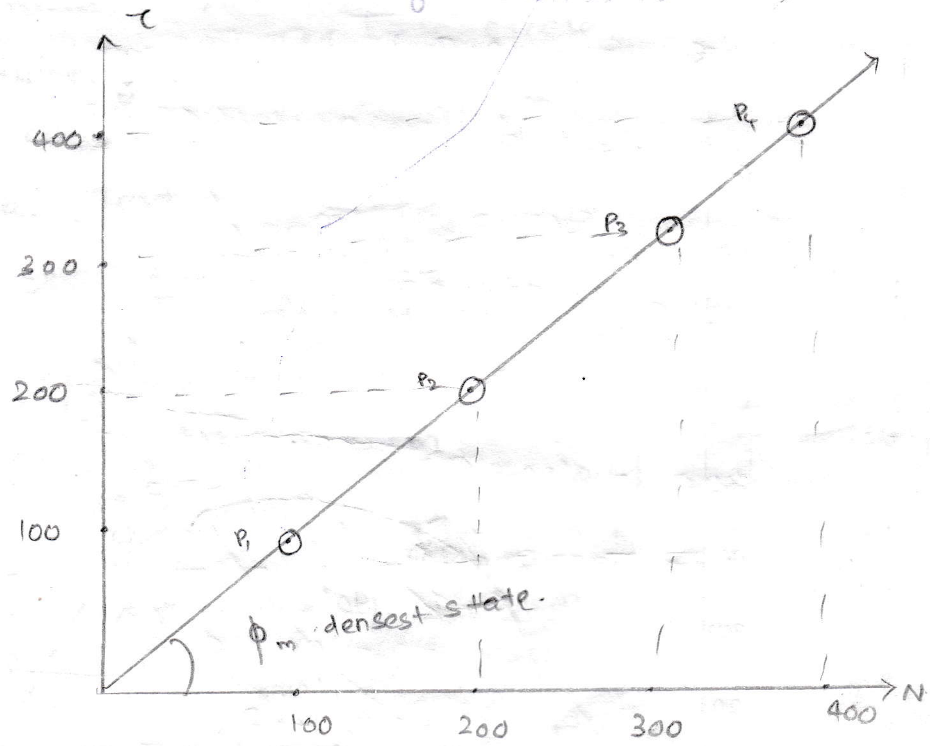
1)



2) Determine the angle of shearing resistance of the sand in the dense state.

i) Densest state

ii) loosest state.



Direct shear stress :

3. A specimen of clean dry cohensionless sand is tested in a shear box and the soil failed @ a shear stress of 40 kN/m². when the normal load of the specimen was 50 kN/m². Determine the

i) Angle of shearing resistance

ii) The principal stress during the failure

iii) Direction of the principal planes with respect to the direction of the plane of shearing.

Given :

τ @ failure = 40 kN/m²

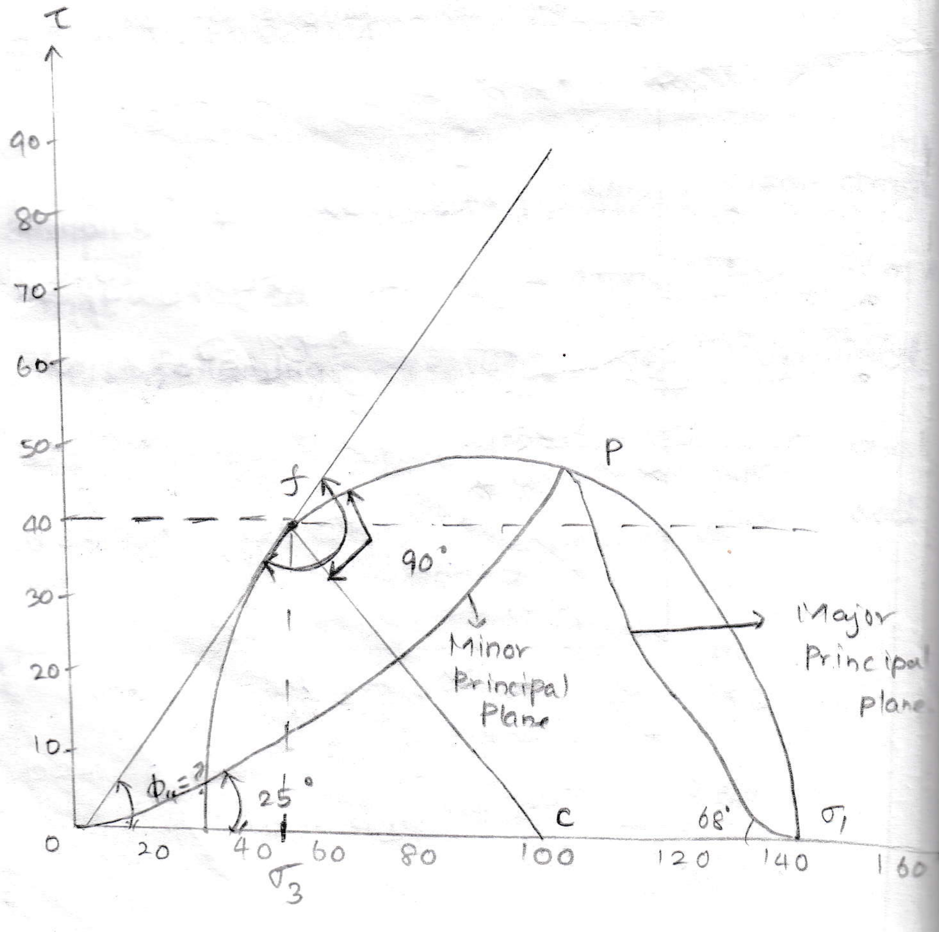
Normal load @ failure = 50 kN/m²

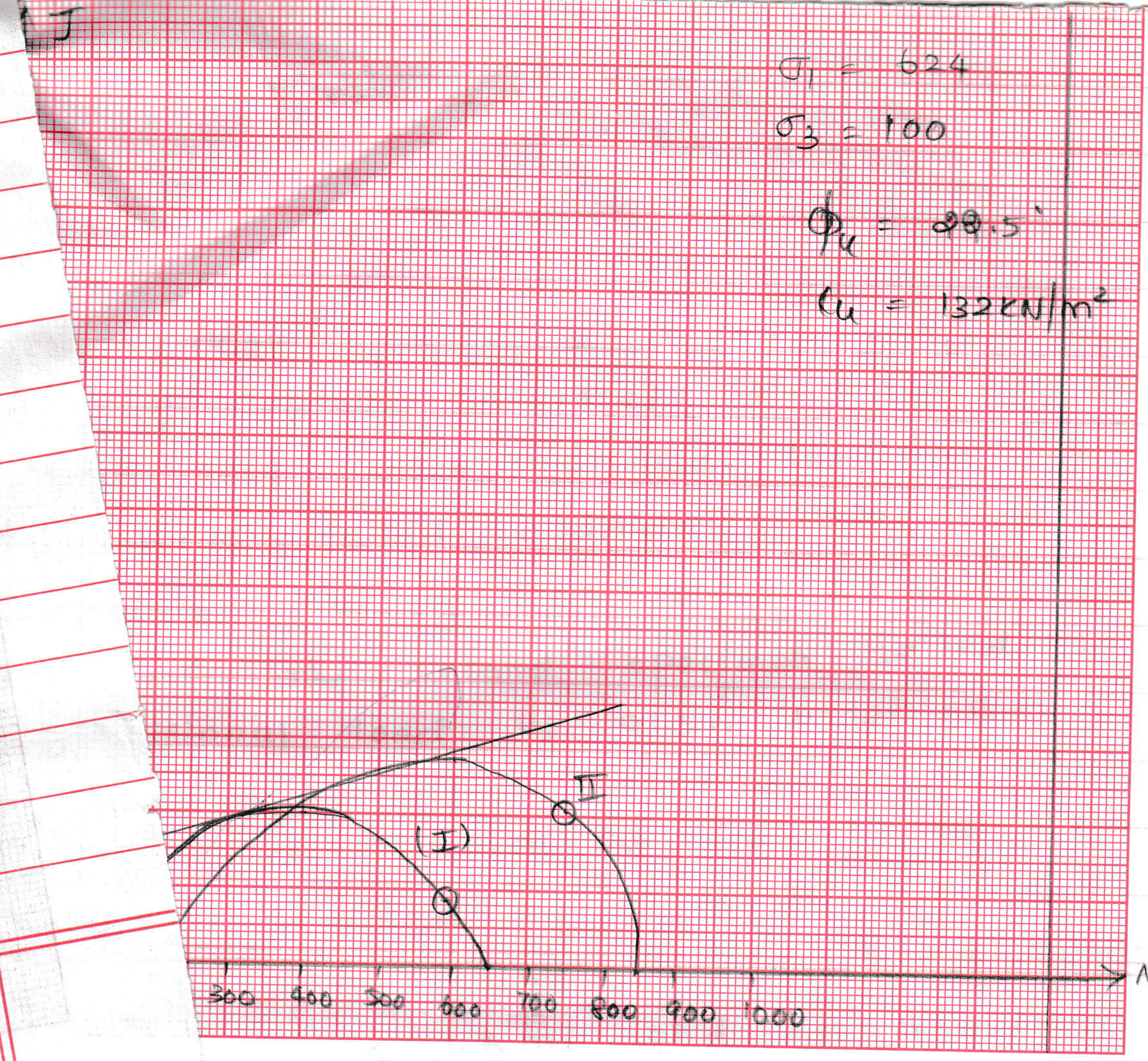
To find :

i) ϕ_u = ?

ii) Principal stress @ failure σ_1 & σ_3

iii) Direction of ~~pressure~~ plane of shearing.





apparent Portion and the angle

resistance i).

Graphically by Mohr's circle.

Analytically:

Triaxial Test:

$$A_2 = \frac{V_1 + \Delta V}{L_1 - \Delta L}$$

First specimen:

$\sigma_3 = 100\ kN/m^2$ Deviator load $\}$ $= 720N$

$d = 4cm$, $h = 8cm$.

$\Delta V = 1.2\,ml = 1.2\ cm^3$ $\Delta L = 0.6cm$.

Solution:

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 4^2}{4} = 12.56\ cm^2$$

$$V_1 = 12.56 \times 8 = 100.48\ cm^3$$

i) $\phi_u = 38.7$

ii) $\sigma_1 = ?$

$\sigma_3 = ?$

26/2

1. Two identical specimen 4 cm in dia and 8 cm height of saptully saturated compacted soil in a triaxial test under undrained condition the first specimen failed additional axial load to an deviator load of 720 N under a self Pressure 100 KN/m^2. The second specimen failed to a additional load of 915 N under a self pressure of 200 kN/m^2 increase in a volume of the first specimen at failure is 1.2 ml and it shorted by 6 cm at failure. the second specimen The increase in volume of and its shortness at failure is 1.6 ml by 8 cm @ failure. Determine the value of apparent portion and the angle of shearing resistance i)

Graphically by Mohr's circle.

Analytically.

Triaxial Test:

$$A_2 = \frac{V_1 + \Delta V}{L_1 - \Delta L}$$

First specimen:

$\sigma_3 = 100\ KN/m^2$ Deviator load $\}$ = 720 N

$d = 4cm$, $h = 8cm$.

$\Delta V = 1.2\ ml = 1.2\ cm^3$ $\Delta L = 0.6cm$.

Solution:

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 4^2}{4} = 12.56\ cm^2$$

$$V_1 = 12.56 \times 8 = 100.48\ cm^3$$

$$A_2 = \frac{100.48 + 1.2}{8 - 0.6} = 13.74\ cm^2$$

$$\text{deviator stress } (\sigma_d) = \frac{720}{13.74} = 52.4\ N/cm^2$$

$$= 524\ kN/m^2$$

$$\sigma_1 = \sigma_3 + \sigma_d$$

$$= 100 + 524$$

$$\sigma_1 = 624\ kN/m^2$$

Second specimen :

$$\sigma_3 = 200\ kN/m^2$$

Deviator load = 915 N

$d = 4\ cm$

$h = 8\ cm$

$\Delta V = 1.6\ ml = 1.6\ cm^2$

$\Delta L = 0.8\ cm$

$$A_1 = \frac{\pi d^2}{4} = \frac{\pi \times 4^2}{4} = 12.56\ cm^2$$

$$\text{deviator stress} = \frac{915}{14.18} = 64.5$$

$$A_2 = \frac{100.48 + 1.6}{8 - 0.8} = 14.18\ cm^2$$

$$\sigma_1 = \sigma_3 + d = 200 + 645.53$$

$$\sigma_1 = 845.3\ kN/m^2$$

ii) Analytical Method :

$$\sigma_1 = \sigma_3 N\phi + 2\,cu\sqrt{N\phi} \rightarrow (1)$$

(I) $624 = 100N\phi + 2cu\sqrt{N\phi}$ ... (2)

II) $845 = 200N\phi + 2cu\sqrt{N\phi}$

$$-221 = -100N\phi$$

$$N\phi = 2.2$$

$$624 = 100(2.2) + 2cu\sqrt{2.2}$$

$$cu = 136\ kN/m^2$$

$$N = \tan^2(45^\circ + \phi/2) \qquad \phi = 22^\circ$$

27/2/14.

1. A vane 10 cm long and 8 cm dia was pressed its soft plane @ the bottom bore. Talk was applied and gradually increased to 45 Nm when failure to place Subsequently the vane rotated rapidly so as to completely removed the soil. The remolded soil was shear at a torque of 18 Nm calculate the cohesion of plane in the Natural and remoulded state, and also find the value of sensitivity.

$$T = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right]$$

$$C = \tau_f$$

Natural State:

$T = 45$ Nm

$H = 10$ cm $= 1 \times 10^{-2}$ m

$d = 8$ cm $= 8 \times 10^{-2}$ m

$$45 = \pi \times (8 \times 10^{-2})^2 \tau_f \left(\frac{1 \times 10^{-2}}{2} + \frac{8 \times 10^{-2}}{6}\right)$$

$\tau_f = 35338.6$ N/m^2

$\tau_f = 35.34$ kN/m^2

Remoulded State:

$T = 18$ Nm

$H = 10$ cm $= 10 \times 10^{-2}$ m.

$d = 8$ cm $= 8 \times 10^{-2}$ m.

$$T = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right]$$

$$18 = \pi (8\times10^{-2})^2 \tau_f \left[\frac{10\times10^{-3}}{2} + \frac{8\times10^{-2}}{6}\right]$$

$$\tau_f = 14135.47 N/m^2$$

$$\boxed{\tau_f = 14.14 kN/m^2}$$

$$\text{Sensitivity} = \frac{C\ \tau_f \text{ of Natural}}{C\ \tau_f \text{ of remoulded}}$$

$$= \frac{35.34}{14.14}$$

$$= 2.5 \text{ cm.}$$

Skempton's Pore Pressure Parameters :

The change in the Pore pressure due to ch[ange] in the applied stress during an undrained she[ar] may be explained in terms of emphrical co-efficient called the pore pressure Paramet[er].

It is a dimensionless number that indicates Fraction of total stress increment that show an exeits pore Pressure for the condition of pore drainage. $\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$

$\Delta\sigma_1 = \gamma \Delta H$ $\quad \Delta\sigma_3 = \frac{1}{2}\Delta\sigma_1$

Effective Stress :

1. An embankment 5m height is Made up of soil whose effective ~~stress~~ Parameters are $c' = 50\,kN/m^2$ and $\phi' = 60°$ and unit weight $\gamma = 16.2\,kN/m^2$ the Pore Pressure Parameters as found from triaxial test or $A = 0.4$; $B = 0.92$. Find the shear strength of the soil @ the base of the embankment Just after the fill has been raised from 5m to 9m; assume that the desipation of Pore Pressure is negligible during this state of construction @ any Point and that the lateral Pressure is one half of the vertical pressure.

Given:

$\gamma = 162\,kN/m^2$

$c' = 50\,kN/m^2$

$\phi = 16°$

$A = 0.4$

$B = 0.92$

$H = 5m$

$\Delta H = 9m - 5m = 4m.$

Find:

Shear strength = ?

$$\Delta u = B\left[\Delta\sigma_3 + A\left(\Delta\sigma_1 - \Delta\sigma_3\right)\right]$$

$$\Delta\sigma_1 = \gamma\,\Delta H = 16.2 \times 4$$

$$= 64.8\,kN/m^2$$

$$\Delta\sigma_3 = 1/2\ \Delta\sigma_1$$

$$= \frac{1}{2} \times 64.8$$

$$= 32.4\,kN/m^2$$