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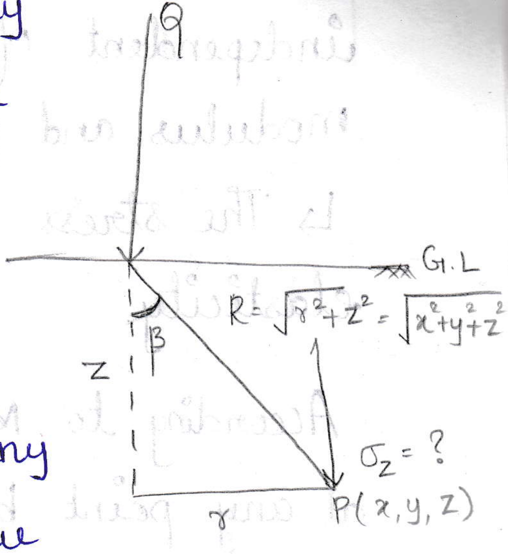
# Unit - 3

## STRESS DISTRIBUTION

A stress distribution at any point below ground surface due to concentrated load plays an important role calculation of consolidation.




A stress distribution at any point below ground surface due to point load has been developed by the following scientist.

1. Mr. Boussinesq — Widly adopted
2. Mr. Westerguard — Mainly for stratified soil.
3. Newmark (Influence chart) — Irregular footing
4. 2:1 method (Approximate method) — Where the slope of stress is 2 vertical : 1 Horizontal

### Assumption :

↳ The applied load is truely vertical and concentrated point load.

↳ The soil mass is weight less.

↳ The soil mass is homogeneous.

↳ The soil mass is isotropic and semi-infinite.

↳ The stress developed in the soil is independent of soil property like young's modulus and poisson's ratio.

↳ The stress is mainly dependent on linear elasticity.

According to Mr. Boussinesq, vertical stress at any point below footing is given by formula,

$$\sigma_z = \frac{Q}{z^2} \times I_B$$

where,

$K_B = I_B$ = Boussinesq influence factor (or) Stress coefficient.

$$I_B = K_B = \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(r/z\right)^2\right)^{5/2}}\right]$$

If, $\frac{r}{z} = 0$

$$I_B = 0.477$$

$$\sigma_z = 0.477 \frac{Q}{z^2}$$

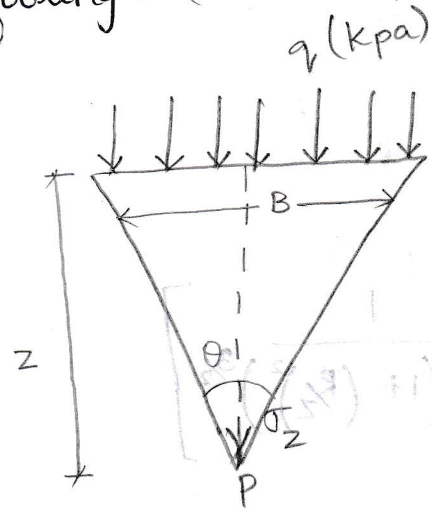
Note :

1. If $r/z$ increases then the vertical stress is decreases.

2. If the value of z increases then the vertical stress is decreases to great extent.

According to Mr. Boussinesq, vertical stress at any point below strip footing, circular footing and for railway line load can be given below,

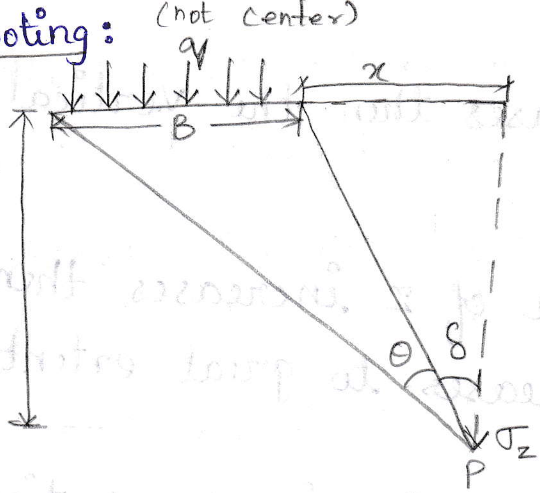
Strip footing : (Center Strip)




The vertical stress ($\sigma_z$) at a point 'P' due to uniform load intensity 'q' on a strip of width 'B' and semi-infinite length is given by

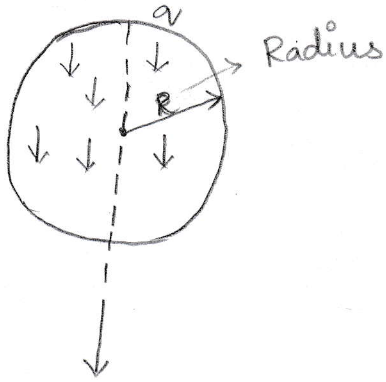
$$\sigma_z = \frac{q}{\pi}\left[\theta + \sin\theta\right]$$

**Strip footing:**



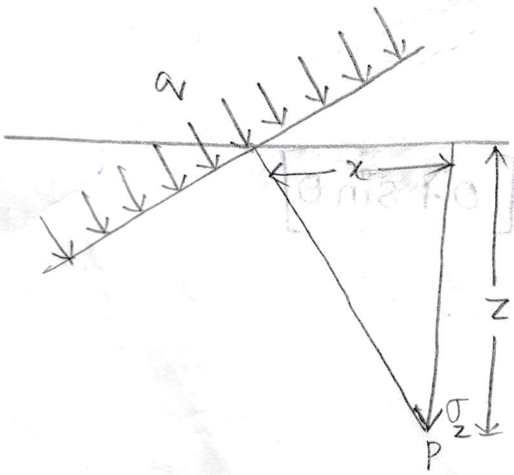

$$\sigma_z = \frac{q}{\pi}\left[\theta + \sin\theta \cdot \cos(\theta + 2\delta)\right]$$

**Circular footing:**




$$\sigma_z = q\left[1 - \frac{1}{\left(1 + (R/z)^2\right)^{3/2}}\right]$$

**Railway Line Load:**




$$\sigma_z = \frac{q}{z} \cdot \frac{2}{\pi}\left[\frac{1}{\left(1 + (x/z)^2\right)^2}\right]$$

According to Mr. Westergard the Vertical stress

According the Mr. Westergard the soil mass is made up of thin sheet which are rigid enough so that only vertical displacement can takes place and lateral displacement is Nil.

$$\sigma_z = \frac{Q}{Z^2} \times I_w$$

where, $I_w$ = Westergard Coefficient.

$$I_w = \frac{1}{\pi}\left[\frac{1}{\left(1+2\left(\frac{r}{z}\right)^2\right)^{3/2}}\right]$$

If, $\frac{r}{z} = 0$

$$I_w = 0.318$$

$$\sigma_z = 0.318\,\frac{Q}{Z^2}$$

Note:

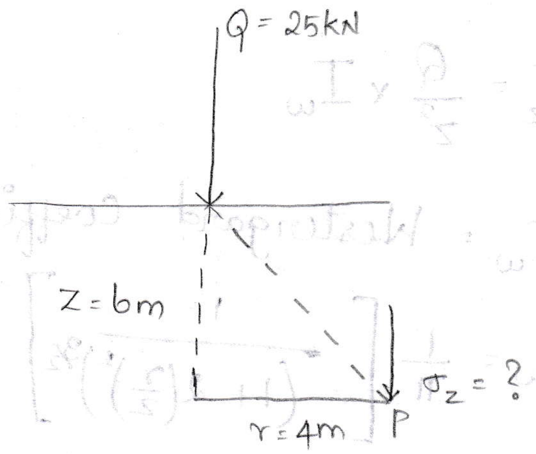
1. If $r/z$ less than 0.8 than the vertical stress given by Westergard i.e., equal to $2/3$ of Boussinesq equation.

2. If $r/z$ less than 0.5 then the vertical stress given by Westergard is always than that of Boussinesq equation.

3. If $r/z$ more than 1.5 then the vertical stress given by Westergard is more than that of Boussinesq equation.

Problems:

1. A 25 kN point load acts on the surface of an infinite elastic medium. Determine the vertical pressure at a point 6m below and 4m away from the load.




Given:

Q = 25 kN

z = 6m

r = 4m

Boussin's Formula:

$$\sigma_z = \frac{Q}{z^2} \times I_B$$

$$I_B = \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(\frac{r}{z}\right)^2\right)^{5/2}}\right]$$

Solution:

$$I_B = \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(\frac{4}{6}\right)^2\right)^{5/2}}\right]$$

$$\therefore I_B = 0.1904$$

$$\sigma_z = \frac{25}{(6)^2} \times 0.1904$$

$$\therefore \sigma_z = 0.1322 \ KN/m^2$$

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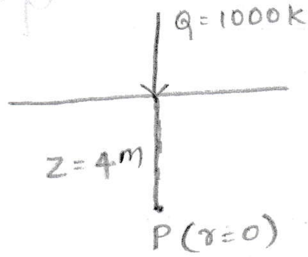
2. A concentrated load of 1000 KN is applied at the ground surface. Compute the vertical pressure at a depth of 4m below the load.




Solution:

Given: $Q = 1000 KN$

$Z = 4m$

$r = 0$

Boussin's Formula,

$$\sigma_z = \frac{Q}{Z^2} \cdot K_B$$

$$K_B = \frac{3}{2\pi}\left[\frac{1}{(1+(r/z)^2)^{5/2}}\right]$$

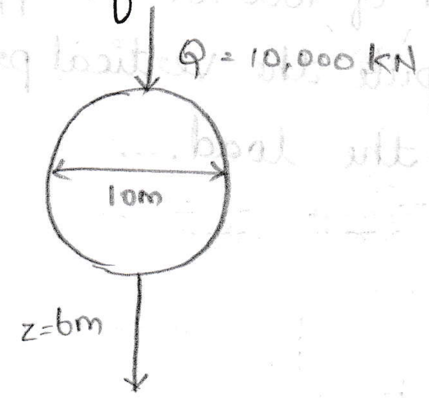
$$= \frac{3}{2\pi}\left[\frac{1}{(1)^{5/2}}\right]$$

$$\therefore K_B = 0.477$$

$$\sigma_z = \frac{1000}{(4)^2} \times 0.477$$

$$\therefore \sigma_z = 29.81 \ KN/m^2$$

3. A circular water tank has a circular footing dia is 10m and the total weight of tank is 10,000 kN, below the foundation at a depth of 6m there is a weak satum having bearing capacity is 150 kPa. At certain condition whether the given size of footing is safe or not.




$Q = q \times Area$

Solution :

$$\sigma_z = q\left[1 - \frac{1}{\left(1+\left(R/z\right)^2\right)^{3/2}}\right]$$

$$Q = q \times Area$$

$$q = \frac{Q}{Area}$$

$$= \frac{10,000}{\frac{\pi(10)^2}{4}}$$

$\therefore q = 127.32$ kpa or $kN/m^2$

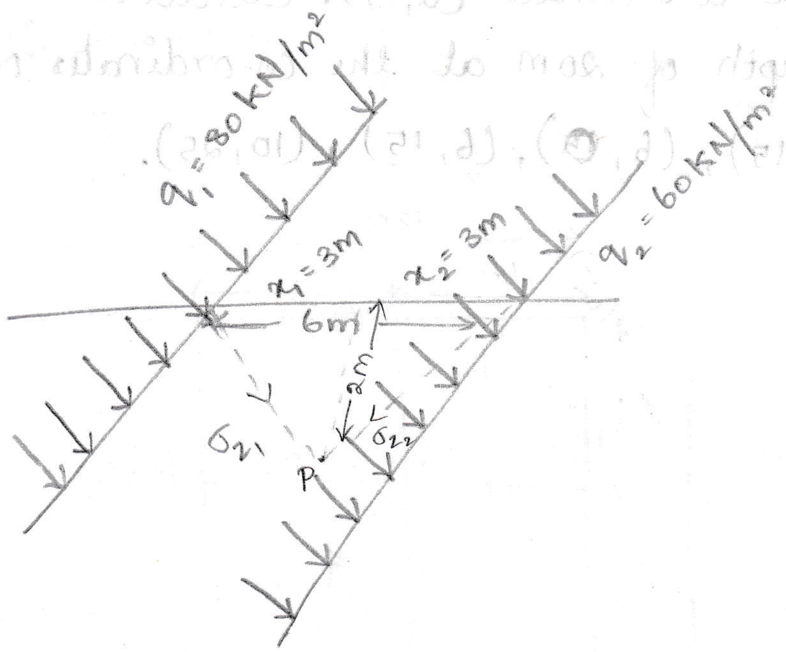
$$\sigma_z = 127.32\left[1 - \frac{1}{\left(1+\left(5/6\right)^2\right)^{3/2}}\right]$$

$\therefore \sigma_z = 69.6$ kPa

$\sigma_z = 69.6 < 150$ kPa

Given size of footing is safe.

4. Two railway lines are separated at a distance of 6m centre to centre. The intensity of load on one track is 80 kN/m and have another track 60 kN/m². Determine vertical pressure at the halfway between two tracks. Take depth below the track is 2m.




Solution:

$$\sigma_{z_1} = \frac{q_1}{z} \times \frac{2}{\pi}\left[\frac{1}{\left[1+\left(x/z\right)^2\right]^2}\right]$$

$$= \frac{80}{2} \times \frac{2}{\pi}\left[\frac{1}{\left(1+\left(3/2\right)^2\right)^2}\right]$$

$$\therefore \sigma_{z_1} = 2.41 \text{ kN/m}^2$$

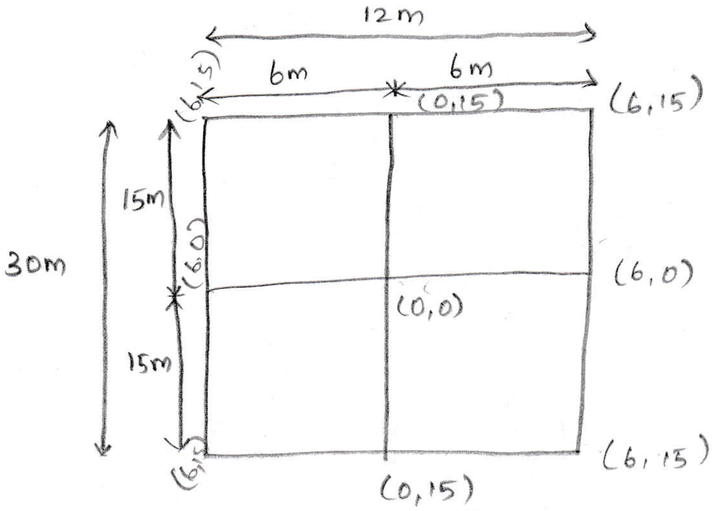
$|||^{ly}$ $\sigma_{z_2} = 1.82 \text{ kN/m}^2$

$$\sigma_z = \sigma_{z_1} + \sigma_{z_2}$$

$$= 2.41 + 1.82$$

$$\therefore \sigma_z = 4.23 \text{ kN/m}^2$$

5. Rectangular raft of size 30x12 m founded on a ground surface is subjected to an uniform pressure of 150 kPa. Assume the centre of area is the origin of co-ordinates (0,0) and corners have co-ordinate (6,15). Calculate stresses at a depth of 20m at the co-ordinates of (0,0), (0,15), (6,0), (6,15), (10,25).




Solution:

At Co-ordinates (0,0): $z = 20m$

$$\sigma_z = \frac{Q}{z^2}\left[\frac{3}{2\pi}\left[\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]}\right]^{5/2}\right]$$

$Q = q \times area$

$= 150 \times 30 \times 12$

$\therefore Q = 54000$ kN

$r = \sqrt{x^2+y^2}$

Here, $x=0$ and $y=0$

$\therefore r=0$

$$\sigma_z = 0.477\frac{Q}{z^2}$$

$$= 0.477 \times \frac{54000}{(20)^2}$$

$$\therefore \sigma_z = 64.395 \text{ kN/m}^2$$

At coordinates (0, 15):

$$r = \sqrt{x^2+y^2}$$

$$= \sqrt{0^2+15^2}$$

$$\therefore r = 15\text{m}$$

$$\sigma_z = \frac{54000}{(20)^2} \times \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(15/20\right)^2\right)^{5/2}}\right]$$

$$\therefore \sigma_z = 21.122 \text{ kN/m}^2$$

At co-ordinates (6, 0):

$$r = \sqrt{6^2+0^2}$$

$$\therefore r = 6\text{m}$$

$$\sigma_z = \frac{54000}{(20)^2} \times \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(6/20\right)^2\right)^{5/2}}\right]$$

$$\therefore \sigma_z = 51.96 \text{ kN/m}^2$$

At coordinates (6, 15):

$$r = \sqrt{6^2+15^2}$$

$$\therefore r = 16.15$$

$$\sigma_z = \frac{54000}{(20)^2} \times \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(\frac{16.15}{20}\right)^2\right)^{5/2}}\right]$$

$$\therefore \sigma_z = 18.362 \text{ kN/m}^2$$

At co-ordinates (10, 25):

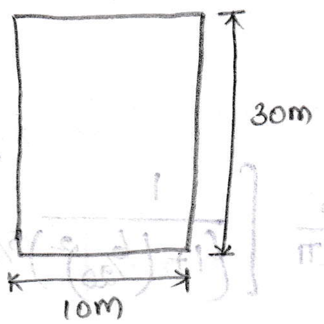
$$r = \sqrt{10^2+25^2}$$

$$\therefore r = 26.9 \text{ m}$$

$$\sigma_z = \frac{54000}{(20)^2} \times \frac{3}{2\pi}\left[\frac{1}{\left(1+\left(\frac{26.9}{20}\right)^2\right)^{5/2}}\right]$$

$$\therefore \sigma_z = 4.86 \text{ kN/m}^2$$

6. Raft foundation has size 10 × 30 m is subjected to load of 150 kpa. Determine the vertical stress at the centre of footing at a depth of 20 m. If this footing is connected into circular footing bearing same area, then what will be the vertical stress.




Z = 20m

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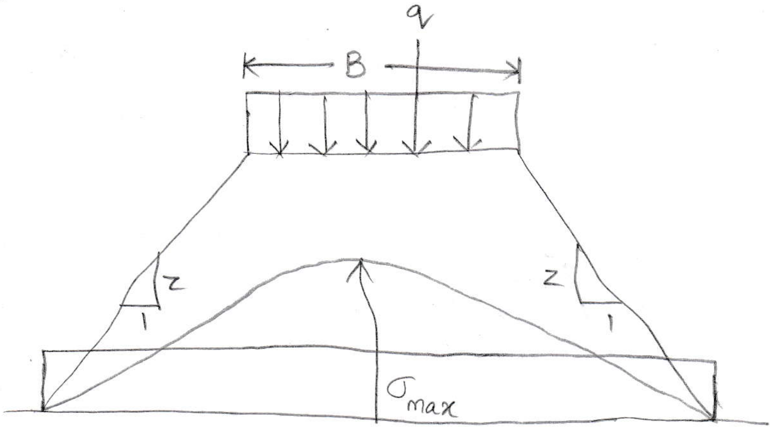
Z : 1 method :




For a rectangular footing,

$$\sigma_z = \frac{q(L \times B)}{(B+z)(L+z)}$$

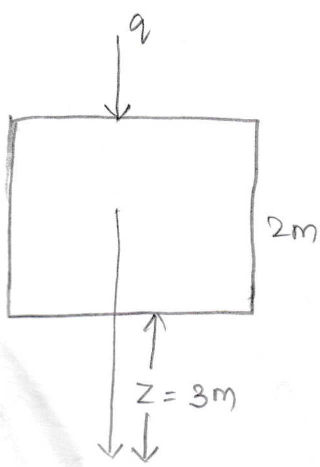
For a square footing,

$$\sigma_z = q\frac{B^2}{(B+z)^2}$$

For a strip shaped footing,

$$\sigma_z = q\frac{B}{(B+z)}$$

7. A square footing having size 2x2 m is subjected to load 100 KPa. Determine the vertical stress 3m below the center by using Z:1 method.



Given:

$B = 2m$

$z = 3m$

$q = 100 \, kpa$

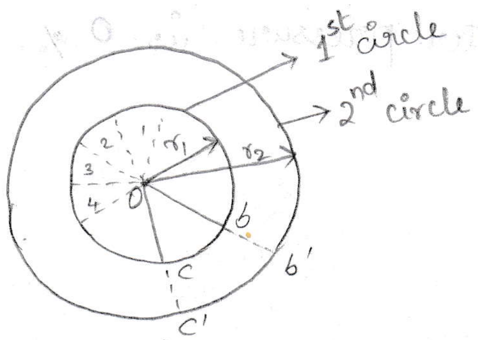
Solution:

$$\sigma_z = q \frac{B^2}{(B+z)^2}$$

$$= 100 \frac{2^2}{(2+3)^2}$$

$$\therefore \sigma_z = 16 \, kpa$$

Newmark Influence Chart: (Derivation refer book)




$$\frac{\sigma_z}{20} = \frac{q}{20}\left[1 - \left(\frac{1}{1+(r/z)^2}\right)^{3/2}\right]$$

$$\frac{\sigma_z}{20} = 0.005q \text{ (arbitary fixed value)}$$

$$\frac{\sigma_z}{20} = 0.005q = \frac{q}{20}\left[1 - \left(\frac{1}{1+(r_1/z)^2}\right)^{3/2}\right]$$

$$\frac{r_1}{z} = 0.270$$

$$\therefore r_1 = 0.270z$$

General equation,

$$\sigma_z = N \times q \times 0.005$$

Consolidation:

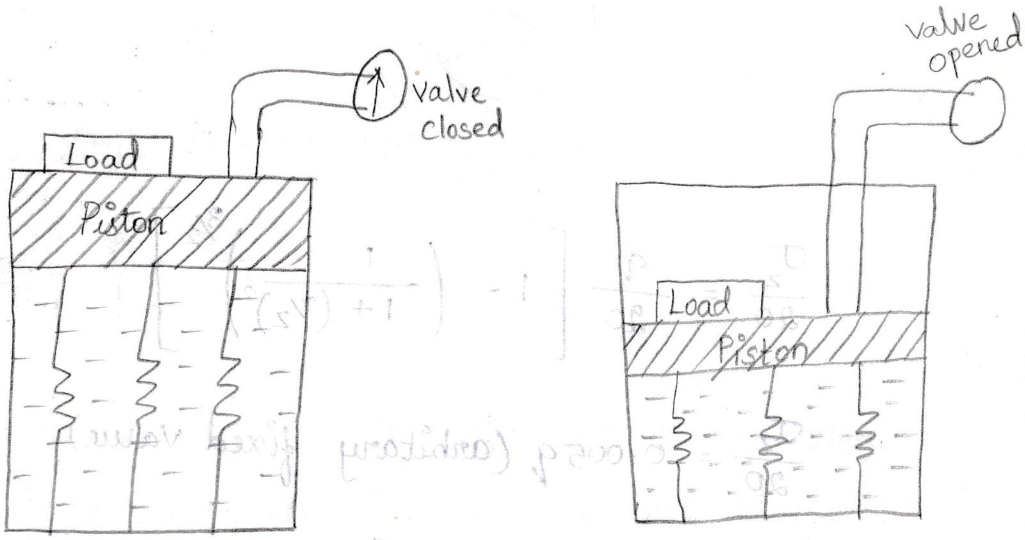
↳ removal of water in soil.

According to Terzaghi, every process involving a decrease in the water content of a saturated soil without replacement of the water by air is called a process of consolidation.

Primary consolidation is a slow process which takes place due to long term static load.

%
2m

- If pore water pressure is 100% then consolidation will be 0%.
- If pore water pressure is 0 % then consolidation will be 100%.




Spring analogy or mechanical analogy

Terzaghi, demostrated the consolidation with the help of cylinder and piston are called spring analogy.

- If the valve of drainage is closed, then the piston will not slide down representing that water is incompressible and total applied load is resisted the water only.

• If the value of drainage is open, then the piston is slide down due to which water is drained out because of load transfer to spring and hence consolidation takesplace.

Technical term:

1. Co-efficient of compressibility ($a_v$):

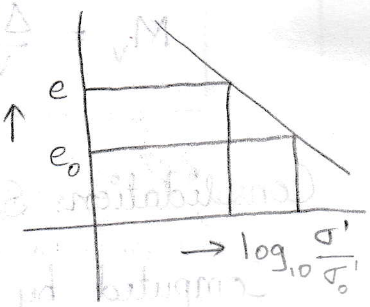
$$a_v = \frac{-\Delta e}{\Delta \sigma'} = \frac{e_0 - e}{\sigma' - \sigma_0'} \text{ in } m^2/kN$$

where, $e_0$ = Initial void ratio corresponding to initial pressure ($\sigma_0'$)

$e$ = Void ratio of increased pressure ($\sigma'$)

2. Compression Index ($C_c$):

$$C_c = \frac{e_0 - e}{\log_{10} \frac{\sigma'}{\sigma_0'}} = \frac{-\Delta e}{\log_{10} \frac{\sigma'}{\sigma_0'}}$$




It represents the slope of the linear portion of the pressure void ratio curve and remains constant with in a fairly large rate of pressure.

NCC Clay (Field): Dr. Terzaghi

$C_c = 0.009$ (LL = 10%)

Remoulded Clay (Lab): Mr. Skempton

$C_c = 0.007$ (LL = 10%)

3. Co-efficient of volume compressibility ($M_v$):

$$\boxed{M_v = \frac{-\Delta e}{1+e_0} \cdot \frac{1}{\Delta\sigma'}}$$

We know that,

$$a_v = -\frac{\Delta e}{\Delta\sigma'}$$

$$M_v = \frac{a_v}{1+e_0}$$

$$\boxed{M_v = \frac{\Delta H}{H} \frac{1}{\Delta\sigma'}}$$

where, $\Delta H$ = Change in thickness

$H$ = Original thickness

$$\boxed{M_v = \frac{\Delta V}{V} \cdot \frac{1}{\Delta\sigma'}}$$

Consolidation Settlement:

Computed by 2 method,

1. Using co-efficient of volume change ($M_v$):

$$\boxed{P_f = M_v H \Delta\sigma'}$$

where, $P_f$ = Final settlement

The consolidation settlement $P_F$, when the soil stratum of thickness 'H' has fully consolidation under a pressure increament $\Delta\sigma'$ in given above.

2. <u>Using</u> <u>Void</u> <u>ratio</u>:

1. $P_f = \Delta H = \dfrac{e_0 - e}{1+e_0}$

2. Normally consolidated soil:

$$\boxed{P_f = H \frac{C_c}{1+e_0} \log_{10} \frac{\sigma'}{\Delta\sigma'}}$$

where, $\sigma' = \sigma_0' + \Delta\sigma'$

$H$ = Soil thickness

$e_0$ = Initial void ratio

$C_c$ = Compression Index

3. Pre consolidated soil:

$$\boxed{i_f = \frac{C_s}{1+e_0} H \log_{10} \frac{\sigma'}{\sigma_0'}}$$

where, $\sigma' = \sigma_0' + \Delta\sigma'$

$C_s$ = Swelling Index

8. Two can specimen A and B of the thickness 2cm and 3cm have equilibrium void ratio 0.68 and 0.72 respectively. Under a pressure of 200 $KN/m^2$. If the equilibrium void ratio of the two soils reduced to 0.50 and 0.62 respectively, when the pressure was increased to 400 $KN/m^2$. Find the ratio of the coefficient of volume compressability of the two specimen.

Given:

Soil specimen A:

A

$H = 2\,cm$

$e_0 = 0.68$

$\sigma_0' = 200\,kN/m^2$

$e = 0.50$

$\sigma' = 400\,kN/m^2$

Soil specimen B:

B

$H = 3\,cm$

$e_0 = 0.72$

$\sigma_0' = 200\,kN/m^2$

$e = 0.62$

To find:

$$\frac{M_{v(A)}}{M_{v(B)}} = ?$$

Solution:

Soil Specimen A:

$$M_v = \frac{-\Delta e}{1+e_0} \cdot \frac{1}{\Delta\sigma'}$$

Change in pressure:

$$\Delta\sigma' = \sigma' - \sigma_0'$$

$$= 400 - 200$$

$$\therefore \Delta\sigma' = 200\,kN/m^2$$

Soil A $(-\Delta e) = e_0 - e$

$$= 0.68 - 0.50$$

$$\therefore -\Delta e = 0.18$$

$$M_{v(A)} = \frac{-\Delta e}{1+e_o} \cdot \frac{1}{\Delta\sigma'}$$

$$= \frac{0.18}{1+0.68} \times \frac{1}{200}$$

$$\therefore M_{v(A)} = 5.35 \times 10^{-4} \ m^2/kN$$

Soil Specimen B:

$$M_v = \frac{-\Delta e}{1+e_o} \cdot \frac{1}{\Delta\sigma'}$$

$$\text{Soil (B)} = -\Delta e = e_o - e$$

$$= 0.72 - 0.62$$

$$\therefore -\Delta e = 0.1$$

$$M_{v(B)} = \frac{-\Delta e}{1+e_o} \cdot \frac{1}{\Delta\sigma'}$$

$$= \frac{0.1}{1+0.72} \cdot \frac{1}{200}$$

$$\therefore M_{v(B)} = 2.906 \times 10^{-4} \ m^2/kN$$

$$\frac{M_{v(A)}}{M_{v(B)}} = \frac{5.35 \times 10^{-4}}{2.906 \times 10^{-4}}$$

$$\therefore \boxed{\frac{M_{v(A)}}{M_{v(B)}} = 1.8404}$$

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9. A water tank is supported by a ring foundation have a outer dia 10 m and inner dia 7.5 m the ring foundation transmit uniform load intensity of 160 KN/$m^2$. Compute the vertical stress induced at a depth of 4 m below the center of ring foundation using Boussineq equation and Westergaard equation.

10. A base of a tower consist of an equilateral triangular frame on the corners of which the three legs of tower is supported. The total weight of the tower is 600 KN which is equally carried by all the three legs. Compute increase in the vertical stress in the soil at a point 5 m below one of the legs.

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